An improved helical subgrid-scale model and large-eddy simulation methods in helical turbulence

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Abstract

For helical isotropic turbulence, an improved two-term helical subgrid-scale (SGS) model is proposed and four types of dynamic methods are given to do large-eddy simulation (LES), which include the standard dynamic procedure, the least quatratic sum dynamic procedure, the dynamic procedure with single constraint of helicity dissipation and the dynamic one with dual constraints of energy and helicity dissipation. Tested *a priori* and *a posteriori* in both steady and decaying helical isotropic turbulence, the four types of dynamic helical models and the dynamic Smogorinsky model are compared with results of direct numerical simulations (DNS) together. Numerical results demonstrate that the three new types of dynamic helical models predict energy and helicity evolution better than the standard dynamic helical model, and the two constrained helical models predict the energy and helicity dissipation rates better than other models. Furthermore, the constrained helical models have higher correlation to the real SGS stress and have more similar probability density functions (PDF) to the DNS results. In general, the two constrained helical models show some more attractive features than other models.

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I. INTRODUCTION

In turbulent flows, helicity is an important physical quantity. It is widespread in the motions of the atmosphere, ocean circulation and other natural phenomena, and also found in leading edge and trailing vortices shed from wings and slender bodies [1–3]. Helicity, a pseudoscalar quantity, can be defined as $h = \boldsymbol{u} \cdot \boldsymbol{\omega}$, where $\boldsymbol{u}$ and $\boldsymbol{\omega}$ are the velocity and vorticity of the turbulent flows, respectively. Similar to the status of energy in the dynamics of ideal fluids, helicity has the character of inviscid invariance. This physical property determines that helicity is an important quantity in turbulence research.

Recently, researches on helical turbulence have been considerably forwarded to the fields of theories, experiments and numerical simulations. Based on the helical decomposition of velocity, the mechanism of existing a joint forward cascade of energy and helicity has been explained in theory [4]. Cascades existing in helical turbulence have space scale and time scale [5–8], and the researches showed that the existing space scale of helicity cascade was larger than energy cascade. In the inertial range, the joint cascade of energy and helicity was dominated by the energy cascade time scale in the low wave number and the helicity cascade time scale in high wave number. Using direct numerical simulation of helical isotropic turbulence, energy and helicity flux were studied. It was shown that helicity flux was more intermittent than the energy flux and the spatial structure was much finer [9].

Large-eddy simulation, as an important method, has been widely used to research turbulent flows. Several kinds of SGS models [10–18] have been proposed so far, such as eddy-viscosity model, dynamic model, vortex model, *et al.* Now, among these SGS stress models, the dynamic mixed model is used most often. Usually, the standard dynamic mixed models can not predict the energy dissipation properly [19]. Some researches suggested that it would be much better if some physical constraints were taken into account [20–22]. Specially, a constrained subgrid-scale stress model was proposed in homogeneous isotropic turbulence recently [23], and in view of this constraint, the numerical results were improved greatly. The constrained condition of this model fit the physical constraint of energy dissipation.

In helical isotropic turbulence, there exists a joint energy and helicity cascades. Thinking of the SGS helicity dissipation rate, Y. Li *et al.* [24] have proposed a two-term helical SGS model as

$$\tau_{ij}^{mod} = C_1 \Delta^2 |\widetilde{S}| \widetilde{S}_{ij} + C_2 \Delta^3 |\widetilde{S}| \widetilde{R}_{ij}, \tag{1}$$

where $\widetilde{S}_{ij} = \frac{1}{2}\left(\partial_j \widetilde{u}_i + \partial_i \widetilde{u}_j\right)$ is the strain-rate tensor at the grid scale Δ, and $\widetilde{R}_{ij} = \frac{1}{2}\left(\partial_j \widetilde{\omega}_i + \partial_i \widetilde{\omega_j}\right)$

is the symmetric vorticity gradient at Δ. The C_1 and C_2 are two model coefficients.

In this paper, we propose a rectified two-term helical SGS model based on Eq. (1). And at the same time, we also propose three types of LES methods, a new dynamic procedure (the least quadratic sum dynamic procedure), the dynamic procedure with single constraint and the dynamic one with dual constraints. Here, the three types of new dynamic helical models, the standard dynamic helical model and the dynamic Smogorinsky model are tested *a priori* and *a posteriori*. Comparing the results, we can get some beneficial conclusions.

II. THEORETICAL ANALYSIS FOR LES MODEL AND METHODS

A. The improved helical SGS model

In the inertial range of helical turbulence, there exists the assumption of scale-invariance, and it demands the SGS models to fit the assumption. To ensure the second term of Eq. (1) scale invariant in inertial range, we rectify the form of the second term and the new helical SGS model can be expressed as

$$\tau_{ij}^{mod} = C_1\Delta^2|\widetilde{S}|\widetilde{S}_{ij} + C_2\lambda_\Delta^2\Delta|\widetilde{S}|\widetilde{R}_{ij}, \tag{2}$$

where $\lambda_\Delta^2 = 15\langle\widetilde{u_i}\cdot\widetilde{u_i}\rangle/\langle\widetilde{\omega_i}\cdot\widetilde{\omega_i}\rangle$, and $\langle\cdot\rangle$ denotes an average over directions of statistical homogeneous field or over pathlines. Similar to the definition of Taylor microscales, λ_Δ can be defined as Taylor microscales at the scale Δ. In theory, the improved helical model can predict the energy and helicity dissipation rates well simultaneously.

To validate the new helical SGS model *a priori*, a DNS of three-dimensional incompressible homogeneous isotropic helical turbulence is introduced here. It solves the forced N-S equations using a pseudo spectral code in a cubic box with periodic boundary conditions, and the numerical resolution is 512^3. A Guassian random field is the initial flow condition, and it has an energy spectrum as

$$E_0(k) = Ak^2U_0^2k_0^{-5}e^{-\frac{2k^2}{k_0^2}}, \tag{3}$$

where $k_0 = 4.5786$ and $U_0 = 0.715$. The whole system is maintained by a constant energy input rate $\epsilon = 0.1$ and a constant helicity input rate $\eta = 0.3$ in the first two wave number shells. The kinetic viscous $\nu = 0.0006$.

In Fig.1, we present λ_δ^2/δ as a function of δ/ζ for *a priori*, where δ is the filter scale varying in the inertial range, and ζ is the Kolmogrove scale. From Fig. 1, one can find that the numerical

behavior of λ_δ^2/δ tends approximately to a constant in the inertial range. Thus, in the inertial range we have $\lambda_\Delta^2 \sim \Delta$ in the numerical behavior in Eq. (2).

FIG. 1: λ_δ^2/δ distributes with δ/ζ for *a priori*. ζ is the Kolmogrov length scale.

B. The standard dynamic method

Based on the assumption of scale-invariance in the inertial range, the standard dynamic models are widely used in large-eddy simulation. The model coefficients are scale-invariant [25] and determined by a dynamic procedure which is due to the Germano identity [14]. The Germano identity can be expressed as

$$L_{ij} = T_{ij} - \overline{\tau}_{ij} = \overline{\widetilde{u}_i \widetilde{u}_j} - \overline{\widetilde{u}}_i \overline{\widetilde{u}}_j, \tag{4}$$

where L_{ij} is the resolved stress, $\tau_{ij} = \widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j$ is the SGS stress at the filter scale Δ. $T_{ij} = \overline{\widetilde{u_i u_j}} - \overline{\widetilde{u}}_i \overline{\widetilde{u}}_j$ is the SGS stress at the test filter scale $\alpha\Delta$ ($\alpha \neq 1$). Note that Eq. (2) is the expression of τ_{ij}^{mod} , and T_{ij}^{mod} can be written as

$$T_{ij}^{mod} = C_1 (\alpha\Delta)^2 |\overline{\widetilde{S}}| \overline{\widetilde{S}}_{ij} + C_2 \lambda_{\alpha\Delta}^2 (\alpha\Delta) |\overline{\widetilde{S}}| \overline{\widetilde{R}}_{ij}. \tag{5}$$

In Eq. (4), L_{ij}, τ_{ij} and T_{ij} are replaced by L_{ij}^{mod}, τ_{ij}^{mod} and T_{ij}^{mod}, and substituting τ_{ij}^{mod} and T_{ij}^{mod} into Eq. (4), we can get

$$L_{ij}^{mod} = T_{ij}^{mod} - \overline{\tau}_{ij}^{mod} = C_1 M_{ij} + C_2 N_{ij}, \tag{6}$$

where

$$M_{ij} = \Delta^2 \overline{|\widetilde{S}| \widetilde{S}_{ij}} - (\alpha\Delta)^2 |\overline{\widetilde{S}}| \overline{\widetilde{S}}_{ij}, \tag{7}$$

$$N_{ij} = \lambda_\Delta^2 \Delta \overline{|\widetilde{S}| \widetilde{R}_{ij}} - \lambda_{\alpha\Delta}^2 (\alpha\Delta) |\overline{\widetilde{S}}| \overline{\widetilde{R}}_{ij}. \tag{8}$$

Here, an average square error is introduced as

$$E^{mod} = \langle (L_{ij} - L_{ij}^{mod})^2 \rangle. \tag{9}$$

By minimizing Eq. (9), the model coefficients C_1 and C_2 can be obtained.

C. The least quadratic sum dynamic method

In helical turbulence, there exists a joint helicity-energy cascade, and we need to consider the energy and helicity dissipation simultaneously. Testing for *a priori*, we find that $\widetilde{S}_{ij}$ and $\widetilde{R}_{ij}$ have a great difference in order of magnitude, and thus the energy dissipation rate $\langle \tau_{ij} \widetilde{S_{ij}} \rangle$ and the helicity dissipation rate $2\langle \tau_{ij} \widetilde{R_{ij}} \rangle$ also have a prominent deviation in order of magnitude.

In order to predict energy and helicity evolution more accurately, we suggest a new type of error as

$$E_N^{mod} = \langle (L_{ij}\overline{\widetilde{S}}_{ij} - L_{ij}^{mod}\overline{\widetilde{S}}_{ij})^2 \rangle + \langle (L_{ij}\overline{\widetilde{R}}_{ij} - L_{ij}^{mod}\overline{\widetilde{R}}_{ij})^2 \rangle. \tag{10}$$

Minimizing Eq. (10), we can get the expression of the model coefficients C_1 and C_2. The new dynamic procedure is the least quadratic sum dynamic procedure.

D. The constrained method

In homogeneous isotropic turbulence, the constrained condition on energy dissipation has been discussed appropriately [23]. In helical turbulence, the constrained conditions need to be decided by the energy dissipation and the helicity dissipation jointly. Fig. 2 shows the energy dissipation ε_δ and helicity dissipation η_δ as a function of δ/ζ. We can see that ε_δ and η_δ are almost constant in the inertial range, and it fits the assumption of scale-invariance in the inertial range. In the inertial

FIG. 2: ε_δ and η_δ distribute with δ/ζ for *a priori*. Line with deltas: ε_δ; Line with squares: η_δ .

range of helical isotropic turbulence, the average energy flux and helicity flux across different scales

are almost invariable and equal to the SGS energy and helicity dissipation respectively,

$$\varepsilon_\Delta = \langle \Pi_E \rangle = -\langle \tau_{ij} \widetilde{S}_{ij} \rangle, \tag{11}$$

and

$$\eta_\Delta = \langle \Pi_H \rangle = -2\langle \tau_{ij} \widetilde{R}_{ij} \rangle, \tag{12}$$

where Π_E and Π_H are the energy and helicity flux through scale Δ, respectively. Substituting τ_{ij}^{mod} for τ_{ij} in Eq. (11) and Eq. (12), we have

$$\varepsilon_\Delta = -\langle \tau_{ij}^{mod} \widetilde{S}_{ij} \rangle, \tag{13}$$

and

$$\eta_\Delta = -2\langle \tau_{ij}^{mod} \widetilde{R}_{ij} \rangle. \tag{14}$$

From Eq. (4) we can know that the energy and helicity dissipation rates at scale are

$$\varepsilon_{\alpha\Delta} = -\langle (L_{ij} + \overline{\tau}_{ij}) \overline{\widetilde{S}}_{ij} \rangle, \tag{15}$$

and

$$\eta_{\alpha\Delta} = -2\langle (L_{ij} + \overline{\tau}_{ij}) \overline{\widetilde{R}}_{ij} \rangle. \tag{16}$$

While the model SGS energy and helicity dissipation rates at scale $\alpha\Delta$ can be expressed as

$$\varepsilon_{\alpha\Delta} = -\langle T_{ij}^{mod} \overline{\widetilde{S}}_{ij} \rangle, \tag{17}$$

and

$$\eta_{\alpha\Delta} = -2\langle T_{ij}^{mod} \overline{\widetilde{R}}_{ij} \rangle. \tag{18}$$

Replacing $\overline{\tau}_{ij}$ with $\overline{\tau_{ij}^{mod}}$ in Eq. (15) and Eq. (16), and then from Eq. (15)-Eq. (18) we can get the two constraints of energy and helicity dissipation rates,

$$\langle T_{ij}^{mod} \overline{\widetilde{S}}_{ij} \rangle = \langle (L_{ij} + \overline{\tau}_{ij}) \overline{\widetilde{S}}_{ij} \rangle, \tag{19}$$

and

$$\langle T_{ij}^{mod} \overline{\widetilde{R}}_{ij} \rangle = \langle (L_{ij} + \overline{\tau}_{ij}) \overline{\widetilde{R}}_{ij} \rangle. \tag{20}$$

The derivations of the constraints Eq. (19) and Eq. (20) are based on the assumption of scale-invariance in the inertial range, and it demands each term of the helical SGS models meets the assumption. If it is true, the following relations are reasonable:

$$\frac{\langle\Delta^2|\widetilde{S}|\widetilde{S}_{ij}\widetilde{S}_{ij}\rangle}{\varepsilon_\Delta}=\frac{\langle(\alpha\Delta)^2|\overline{\widetilde{S}}|\overline{\widetilde{S}}_{ij}\overline{\widetilde{S}}_{ij}\rangle}{\varepsilon_{\alpha\Delta}},\frac{\langle\lambda_\Delta^2\Delta|\widetilde{S}|\widetilde{R}_{ij}\widetilde{S}_{ij}\rangle}{\varepsilon_\Delta}=\frac{\langle\lambda_{\alpha\Delta}^2(\alpha\Delta)|\overline{\widetilde{S}}|\overline{\widetilde{R}}_{ij}\overline{\widetilde{S}}_{ij}\rangle}{\varepsilon_{\alpha\Delta}}, \tag{21}$$

and

$$\frac{\langle\Delta^2|\widetilde{S}|\widetilde{S}_{ij}\widetilde{R}_{ij}\rangle}{\eta_\Delta}=\frac{\langle(\alpha\Delta)^2|\overline{\widetilde{S}}|\overline{\widetilde{S}}_{ij}\overline{\widetilde{R}}_{ij}\rangle}{\eta_{\alpha\Delta}},\frac{\langle\lambda_\Delta^2\Delta|\widetilde{S}|\widetilde{R}_{ij}\widetilde{R}_{ij}\rangle}{\eta_\Delta}=\frac{\langle\lambda_{\alpha\Delta}^2(\alpha\Delta)|\overline{\widetilde{S}}|\overline{\widetilde{R}}_{ij}\overline{\widetilde{R}}_{ij}\rangle}{\eta_{\alpha\Delta}}. \tag{22}$$

Now, we introduce four functions $f_1(\delta)$, $f_2(\delta)$, $h_1(\delta)$ and $h_1(\delta)$, which are

$$f_1(\delta)=\frac{\langle\delta^2|\widetilde{S}|\widetilde{S}_{ij}\widetilde{S}_{ij}\rangle}{\varepsilon_\delta},f_2(\delta)=\frac{\langle\lambda_\delta^2\delta|\widetilde{S}|\widetilde{R}_{ij}\widetilde{S}_{ij}\rangle}{\varepsilon_\delta}, \tag{23}$$

and

$$h_1(\delta)=\frac{\langle\delta^2|\widetilde{S}|\widetilde{S}_{ij}\widetilde{R}_{ij}\rangle}{\eta_\delta},h_2(\delta)=\frac{\langle\lambda_\delta^2\delta|\widetilde{S}|\widetilde{R}_{ij}\widetilde{R}_{ij}\rangle}{\eta_\delta}. \tag{24}$$

In Fig. 3 and Fig. 4 we show the distribution of $f_1(\delta)$, $f_2(\delta)$, $h_1(\delta)$ and $h_1(\delta)$ with δ/ζ. We can see the numerical behavior of the four functions are almost constant in the inertial range, which verifies the assumption of scale-invariance again and the validity of the constrained condition Eq. (19) and Eq. (20).

FIG. 3: (a) $f_1(\delta)$, (b) $f_2(\delta)$ distribute with δ/ζ for *a priori*.

FIG. 4: (a) $h_1(\delta)$,(b) $h_2(\delta)$ distribute with δ/ζ for *a priori*.

III. THE NUMERICAL RESULTS AND ANALYSIS

In this section, we will give *a priori* and *a posteriori* test of the LES models, and do some comparison and analysis. Five SGS models are choosed to compare each other, and they are dynamic Smogorinsky model (DSM), the standard dynamic helical model (DHM), the new dynamic helical model (NDSH), the dynamic helical model with single constraint of helicity dissipation (CDSH1) and the dynamic helical model with dual constraints of energy and helicity dissipation (CDSH2).

First of all, we show some results tested *a priori*. The energy and helicity dissipation rates

FIG. 5: The distribution of SGS energy dissipation rate with δ/ζ for *a priori*. Dashed line: CDSH1; dashdotdot line: CDSH2; line with squares: NDSH; line with deltas: DSH; line with diamonds: DSM; the bold solid line: DNS.

FIG. 6: The distribution of SGS helicity dissipation rate with δ/ζ for *a priori*. Dashed line: CDSH1; dashdotdot line: CDSH2; line with squares: NDSH; line with deltas: DSH; line with diamonds: DSM; the bold solid line: DNS.

are calculated *a priori* in Figs. 5 and 6, respectively. We can see the distribution of energy and helicity dissipation rates in different scales, and in the inertial range the results from CDSH1 and CDSH2 are closer to the DNS result than other models. For energy dissipation rate, the NDSH also gives a better result than DSH and DSM. The results from DSH and DSM are almost the same, and we can draw the conclusion that the second term of DSH has a trivial contribution to the energy dissipation rate. While in Fig. 6, we can see the helicity dissipation rates of NDSH and DSH deviate far from DNS result, which is mainly caused by the second term of the helical model.

In Fig. 7 and Fig. 8, we give the probability density functions (PDF) of the energy flux Π_E

FIG. 7: The distribution of probability density functions for the energy flux at the filter scale Δ for *a priori*. Dashed line: CDSH1; dashdotdot line: CDSH2; line with squares: NDSH; line with deltas: DSH; line with diamonds: DSM; the bold solid line: DNS.

and helicity flux Π_H at scale Δ for the five types of SGS models, respectively. And the PDF from

DNS is also supported here for comparison. Fig. 7 reads that the PDFs of CDSH1 and CDSH2 can predict the backscatters of energy flux, while other models can not. It is shown in Fig. 8 the similar results to the PDFs of energy flux in Fig. 7, and only CDSH1 and CDSH2 can capture the backscatters of helicity flux.

FIG. 8: The distribution of probability density functions for the helicity flux at the filter scale Δ for *a priori*. Dashed line: CDSH1; dashdotdot line: CDSH2; line with squares: NDSH; line with deltas: DSH; line with diamonds: DSM; the bold solid line: DNS.

We show in Fig. 9 the PDFs of the SGS stress weight τ_{12} at scale Δ for different SGS models *a priori*. The PDFs from all the models have the similar trend to the PDF from DNS, and particularly, the PDF from CDSH1 accords well with that from DNS.

FIG. 9: The distribution of probability density functions for the SGS stress weight τ_{12} for *a priori*. Dashed line: CDSH1; dashdotdot line: CDSH2; line with square: NDSH; line with delta: DSH; line with diamond: DSM; the bold solid line: DNS.

To confirm the validity of these SGS models' applying to LES, we use these models to perform three dimensional LES of forced and decaying helical turbulence. It is noting that the resolution of

the LES is 64^3, and the basic filter length is $3\pi/64$. A Gaussian filter is also used here. Resolution of the comparing DNS is 512^3 for both forced and decaying helical turbulence. The kinetic viscous $\nu = 0.0006$, and the Guassian filter is taken here.

In Figs. 10 and 11, we show the steady energy and helicity spectra from the five types of SGS models and DNS. We can see from Fig. 10 that the energy spectra from the five SGS models have no remarkable difference, while from Fig.11 we can see DSH underestimates seriously the helicity spectra close to the grid scale, and DSM and NDSH a little overestimates the helicity spectra close to the grid scale. The CDSH1 and CDSH2 predict both energy and helicity evolution quite well.

FIG. 10: Energy spectra. Bold solid line: DNS; (a) the dashed line: NDSH; line with squares: DSH; line with deltas: DSM (b) the dashed line: CDSH1; line with squares: CDSH2; line with deltas: DSM

FIG. 11: Helicity spectra. Bold solid line: DNS; (a) the dashed line: NDSH; line with squares: DSH; line with deltas: DSM (b) the dashed line: CDSH1; line with squares: CDSH2; line with deltas: DSM

Fig. 12 and Fig. 13 show the time evolutions of the decaying energy and helicity spectra of the SGS models, and they all start from the same fully statistical steady state. In Fig. 12, we can see that CDSH1 underestimates the energy spectra greatly close to the grid scale, and the energy spectra of other models have trivial difference. Helicity is not always positive, which are caused by the helicity's property of pseudoscalar, and thus it is shown in Fig. 13 that the helicity spectra display the character of fluctuations. Also similar to the steady case, the DSH underestimates the helicity close the grid scale, and the results from other models have no obvious difference.

FIG. 12: Energy spectra for decaying helical turbulence (*a posteriori*), at $t = 0$, $6\tau_0$, and $12\tau_0$, where τ_0 is the inertial large eddy turnover time scale. Bold line: DNS; the dashed line: (a) CDSH1, (b) CDSH2, (c) NDSH, (d) DSH, (e) DSM.

FIG. 13: Helicity spectrafor decaying helical turbulence (*a posteriori*), at $t = 0$, $6\tau_0$, and $12\tau_0$. Bold line: DNS; the dashed line: (a) CDSH1, (b) CDSH2, (c) NDSH, (d) DSH, (e) DSM.

IV. CONCLUSIONS

In this paper, we firstly improve the helical SGS model based on an existing helical model to ensure the helical models scale-invariant in the inertial range. Then, we propose a new dynamic method and two types of constrained dynamic methods to predict the energy and helicity dissipation well simultaneously. Using the improved helical SGS model with the three new dynamic methods and the standard dynamic mehod, we have proposed four types of dynamic helical models and compared them with DSM model and DNS.

Through testing for *a priori* and *a posteriori*, we have found that the constrained dynamic helical models predict energy and helicity dissipation rates well and have high correlation with the real SGS stress. At the same time, CDSH1 and CDSH2 can predict the energy and helicity backscatter. NDSH also has some improvement contrasting with DSH, such as predicting the helicity evolution and energy dissipation rate.

In short, the constrained dynamic helical models are quite fit to use in the large eddy simulation of helical isotropic turbulence, and also fit to apply into other systems, such as rotational turbulence and magnetohydrodynamics, *et al*.

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